**Problem Statement Summary:**

Given a dynamic table (array) that **doubles in size** whenever it runs out of space, we are to **find the amortized runtime for inserting nn elements**, using:

1. **Aggregate Method**
2. **Accounting Method**

**a) Aggregate Method**

In the **aggregate method**, we compute the total cost for inserting nn elements and divide it by nn to get the **amortized cost per operation**.

**Cost Breakdown:**

Let’s say the array starts with a capacity of 1 and doubles each time it fills up.

Each insertion either:

* Just inserts the element (cost = 1), or
* Triggers a resize: copy all existing elements to a new array (cost = number of elements copied + 1 for the insertion)

**When resizing occurs:**

| **Resize** | **Size Before** | **Size After** | **Elements Copied** |
| --- | --- | --- | --- |
| 1st | 1 | 2 | 1 |
| 2nd | 2 | 4 | 2 |
| 3rd | 4 | 8 | 4 |
| ... | ... | ... | ... |
| kkth | 2k−12^{k-1} | 2k2^k | 2k−12^{k-1} |

Total number of elements copied over all resizes for nn insertions is:

∑i=0log⁡2n2i=2log⁡2n+1−1=2n−1\sum\_{i=0}^{\log\_2 n} 2^i = 2^{\log\_2 n + 1} - 1 = 2n - 1

**Total Cost:**

* nn insertions (each costs 1)
* ≤2n\leq 2n total copying during resizes

T(n)≤n+2n=3nT(n) \leq n + 2n = 3n

**Amortized Cost per Insertion:**

T(n)n≤3nn=O(1)\frac{T(n)}{n} \leq \frac{3n}{n} = \boxed{O(1)}

**b) Accounting Method**

In the **accounting method**, we assign an **amortized cost per operation** such that we “overcharge” some operations to cover the cost of expensive ones (like resizing).

Let’s assign an **amortized cost of 3** to every insertion.

**For a non-resize insert:**

* Real cost = 1
* Extra 2 units are saved

**For a resize insert:**

* Suppose we insert at size kk, and resizing occurs, moving kk items
* The saved credits (2 from each of the kk previous insertions) cover the cost of copying all kk elements
* The current insert is charged 3: 1 to insert, and 2 credits saved for the future

Thus, **the extra charges cover all resizing costs**, and we always have enough credits.

**Amortized Cost:**

3=O(1)\boxed{3 = O(1)}

**Final Answers:**

**a) Aggregate Method:** Amortized cost = O(1)\boxed{O(1)}  
**b) Accounting Method:** Amortized cost = O(1)\boxed{O(1)}

Let me know if you want the potential method too!